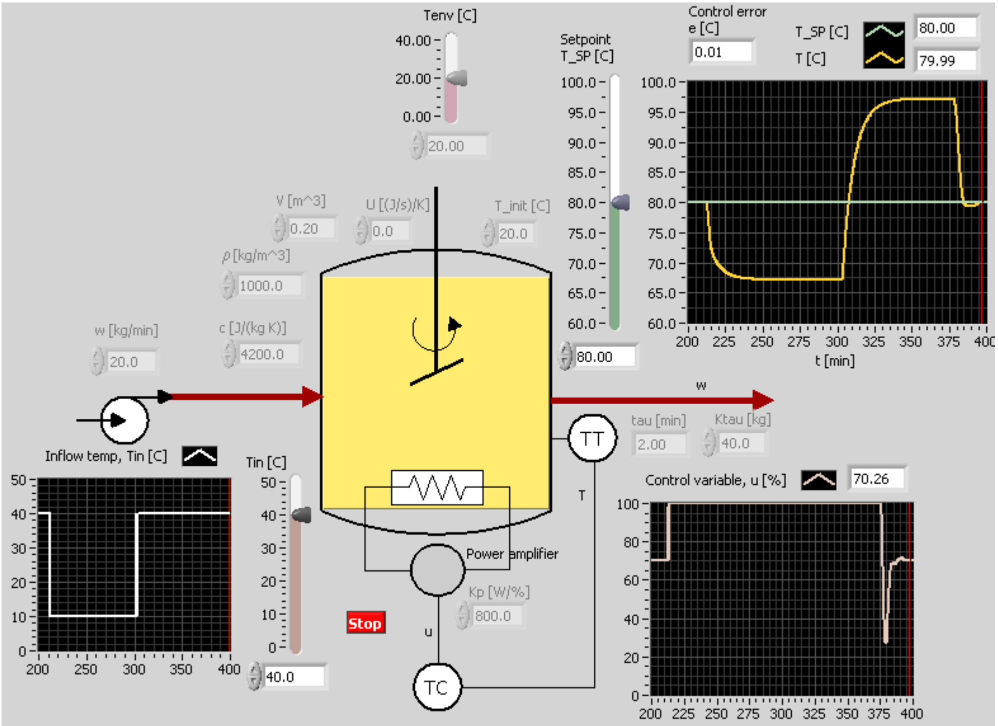

Figure 2.34: Example 2.15: Temperature control *without* anti wind-up

disturbance has changed back to its normal value).

[End of Example 2.15]

2.7.3 Measurement noise. Signal variance

Introduction

You have already seen the problems concerning measurement noise in a control loop, cf. Section2.6.7. Figure 2.22 shows where the measurement noise enters the control loop. Section2.6.7 describes a necessary modification of the derivative term in a PID controller: A lowpass filter is inserted before (in series with) the D-term to attenuate the noise before it is time-differentiated to avoid too large noise-generated responses in the control signal. But what if the lowpass filter in the D-term does not give sufficient noise filtering? Then an additional filter should be included in the feedback path, acting on the measurement signal. This filter can be

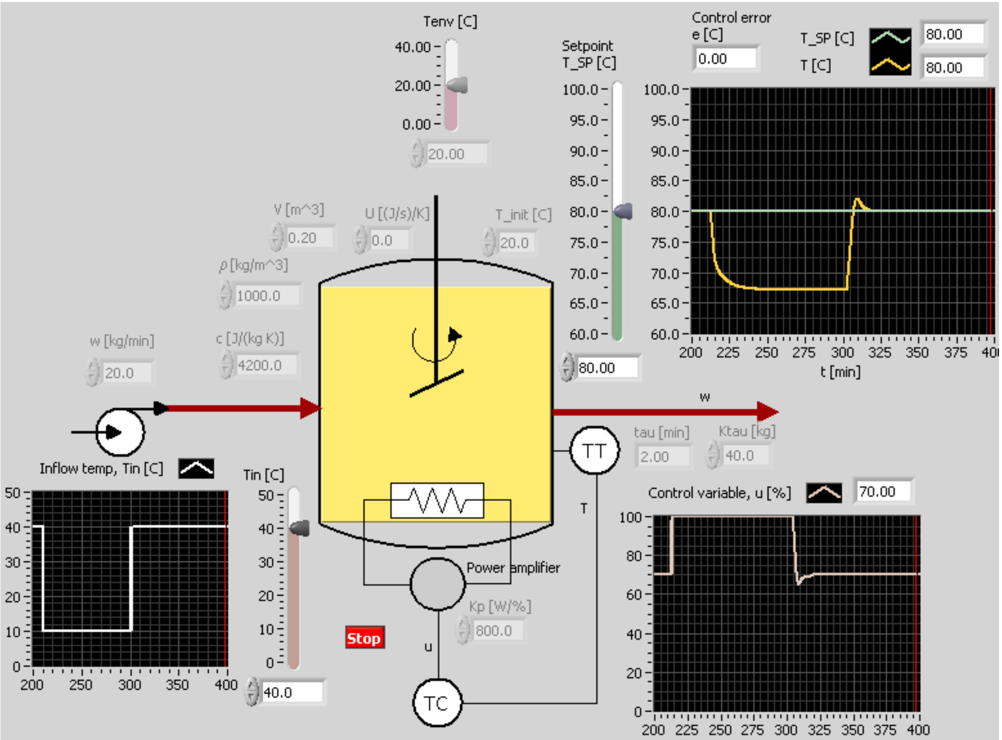

Figure 2.35: Example 2.15: Temperature control *with* anti wind-up

- either a linear dynamic lowpass filter,
- or a deadband filter.

These solutions are described in more detail below.

Calculating the variance

Measurement noise is typically a random signal. The noise propagates through the control system via the controller, causing variations in all variables in the control system. Figure 2.36 shows typical examples of a noisy process measurement and the control variable and in a simulated control system. The variances shown in the figure are calculated as explained below from the 50 most recent samples.

To express the variation of a process variable, the statistical *variance* can be calculated, alternatively the standard deviation which is the square root of the variance. The larger variance, the larger the variations. The

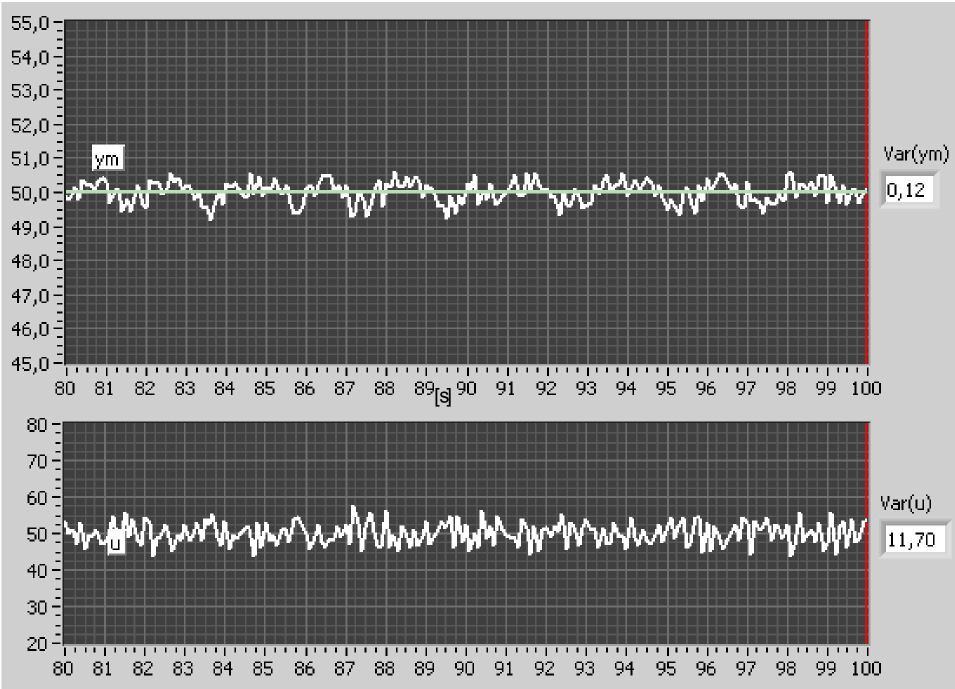

Figure 2.36: Typical examples of a noisy process measurement and the control variable

variance is the mean square deviation about the mean value[11]

$$\mathrm{Var}(y_m) = \frac{1}{N-1}\sum_{k=1}^{N}[y_m(t_k) - m_{y_m}]^2 \tag{2.77}$$

where N is the number of samples and m_{y_m} is the mean value of y_m, which may be calculated by

$$m_{y_m} = \frac{1}{N}\sum_{k=1}^{N} y_m(t_k) \tag{2.78}$$

The numerical value of the variance is usually not particularly useful in itself, but it is useful when comparing signals.

In Example 2.16 variances will be used to express the improvements by using a lowpass filter on the process measurement signal.

[11] To obtain a so-called nonbiased estimate of the variance, you must divide by $N-1$, not by N.

Using a dynamic lowpass filter

Figure 2.37 shows a control loop having a lowpass filter acting on the measurement signal. The filter can be a discrete-time filter implemented in

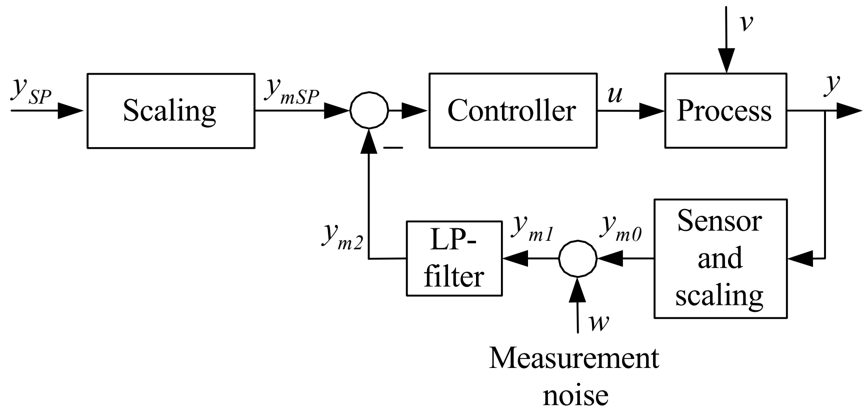

Figure 2.37: Control loop having a lowpass filter acting on the measurement signal. (LP = lowpass.)

the control equipment. It is common that control equipment have inbuilt lowpass filter functions. Alternatively, the filter can be a continuous-time lowpass filter implemented using electronic components external to the control equipment. For example a first order filter can be implemented as an RC-circuit.

Let us assume that the measurement lowpass filter is a first order filter. Such a filter has the following transfer function from filter input x_{in} to filter output x_{out}:

$$\frac{x_{out}(s)}{x_{in}(s)} = H(s) = \frac{1}{\frac{s}{\omega_b} + 1} = \frac{1}{\frac{s}{2\pi f_b} + 1} \tag{2.79}$$

The bandwidth of the lowpass filter is $\omega_b = 2\pi f_b$ where ω_b has unit rad/s and f_b has unit Hz. The bandwidth must be given a value which is smaller than the frequency of the substantial noise frequency components so that these components fall within the stopband of the filter. The bandwidth may be tuned experimentally. Figure 2.38 shows a typical amplitude gain function of first order lowpass filter. One example of a noise frequency component is shown in the figure (it is in the stopband of the filter). The bandwidth is typically defined as the frequency where amplitude gain is $1/\sqrt{2} = 0.71 \approx -3\text{dB}$.

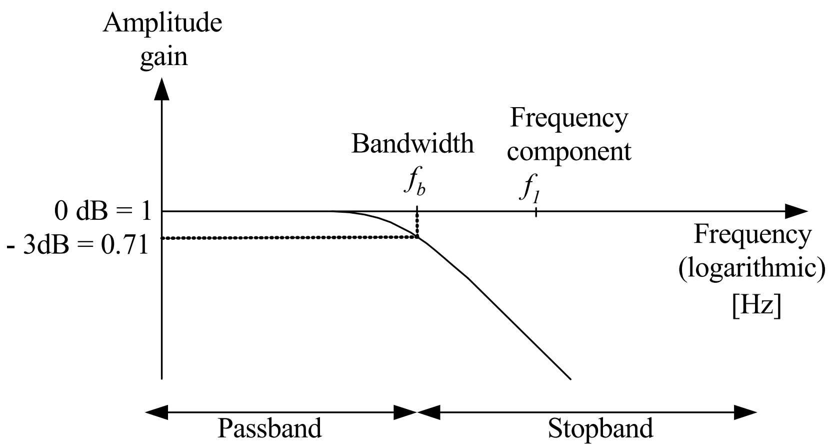

Figure 2.38: Typical amplitude gain function of a lowpass filter. One example of a noise frequency component is shown.

Example 2.16 ***Measurement noise filter in a control loop***

In this example a control system for a process having the following transfer function model is simulated:

$$y(s) = \frac{K_u}{(T_1 s + 1)(T_2 s + 1)} e^{-\tau s} u(s) \qquad (2.80)$$

$$+ \frac{K_v}{(T_1 s + 1)(T_2 s + 1)} e^{-\tau s} v(s) \qquad (2.81)$$

(The process is thus a second order system with time delay.) u is the control variable, and v is the process disturbance. The process parameter are

$$K_u = 1;\ K_v = 1;\ T_1 = 1\text{s};\ T_2 = 0.5\text{s};\ \tau = 0.3\text{s}; \qquad (2.82)$$

The PID parameters are

$$K_p = 2.8;\ T_i = 1.2\text{s};\ T_d = 0.3\text{s}; \qquad (2.83)$$

(tuned with the Ziegler-Nichols' closed loop method). Figure 2.39 shows simulated responses in the control system. The measurement noise is a random signal uniformly distributed[12] between -1 and $+1$. The lowpass filter acts on the process measurement, cf. Figure 2.37. It is switched into the loop at $t = 10$s. From Figure 2.37 we see that the filter removes noise from the process measurement and that the control variable (therefore) is less noisy. The filter is a first order lowpass filter with bandwidth 1.5Hz.

[12] which means that there is equal probability for any value between -1 and $+1$.

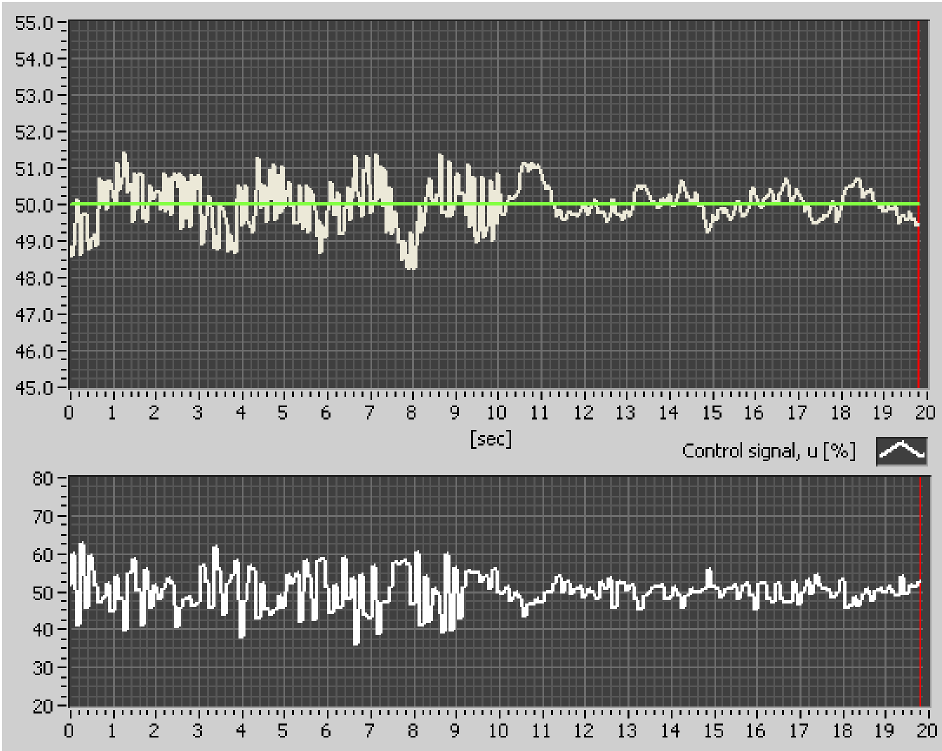

Figure 2.39: Example 2.16: Simulated responses of a control system. A lowpass filter acting on the process measurement signal is switched into the control loop at $t = 10$s.

Table 2.1 shows the variances of the control signal u and the process measurement (after the filter) without and with lowpass measurement filter. The variances are calculated from the 50 most recent signal samples. It is clear from the variances that the filter reduces the influence of the noise in the loop.

Without Filter	**With Filter**
$\mathrm{Var}(u) = 36.4$	$\mathrm{Var}(u) = 5.2$
$\mathrm{Var}(y_m) = 0.33$	$\mathrm{Var}(y_m) = 0.13$

Table 2.1: Variances of control signal u and measurement signal y_m without and with lowpass filter

[End of Example 2.16]

Including a filter in the control loop *changes the dynamic properties* of the loop! Actually, it can cause stability problems in the control loop. In most cases, the less bandwidth (i.e., more sluggish filter), the more reduction of the stability of the loop.

Example 2.17 ***Poor stability because of measurement filter***

Figure 2.40 shows simulated responses for the same control system simulated in Example 2.16. Before $t = 10$s there is no lowpass filter in the loop, while after $t = 10$s a first order lowpass filter with bandwidth 0.2Hz is switched into the loop. The filter causes the control system to have very poor stability.

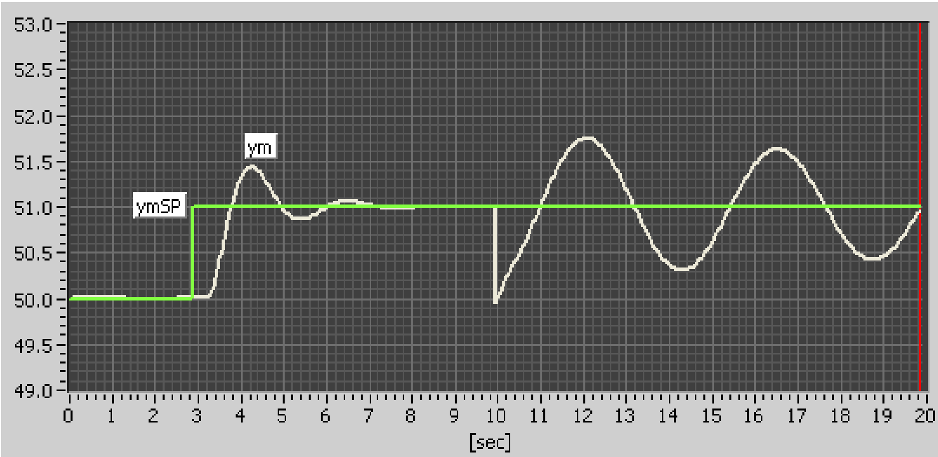

Figure 2.40: Example 2.17: A first order lowpass filter is switched into the control loop at $t = 10$s, causing the control system to have poor stability.

[End of Example 2.17]

If a measurement filter results in poor stability of the control loop – how can that problem be avoided? By *tuning (or re-tuning) the controller with the filter in the control loop.*

It is tempting to select a very small bandwidth of the measurement lowpass filter to achieve strong attenuation of the measurement noise. But in addition to attenuating noise, also frequency components in the ideal (noise-free) process output signal is attenuated. In other words: Important process information may be removed from the measurement signal. This in turn may cause the controller to calculate the control signal on basis of an erroneous control error value. One way to solve this problem, is to introduce a similar filter in series with the setpoint, as shown in Figure . This solution is equivalent to placing one filter in series with (or before) the PID controller in Figure 2.41. A setpoint filter implies that the setpoint which the controller observes, becomes more sluggish since high frequency components are attenuated.

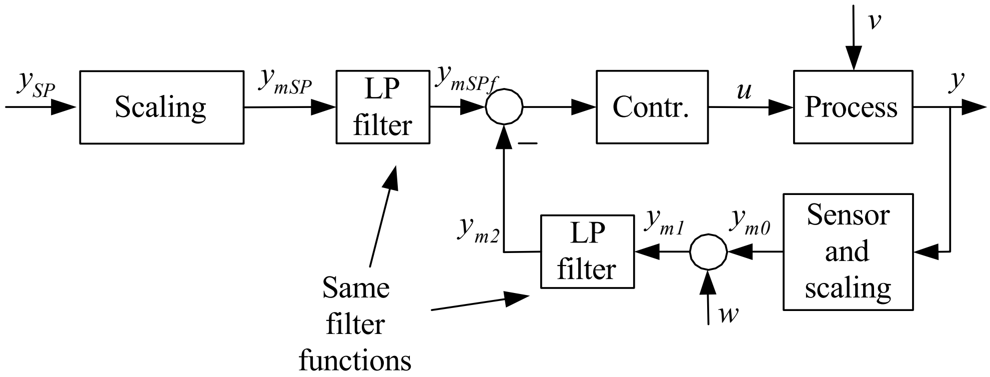

Figure 2.41: Lowpass filter acting on the setpoint

Using a deadband filter

If you know the maximum amplitude of the measurement noise, the noise can be removed from the noisy measurement signal by letting the signal pass through a deadband filter, see Figure 2.42. The output value of the

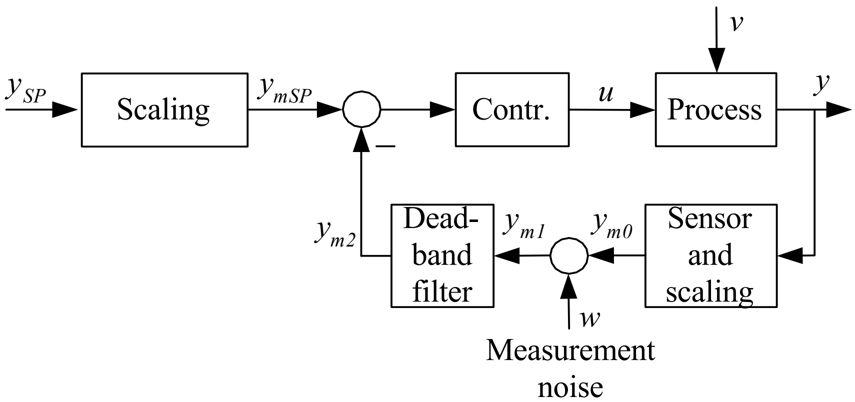

Figure 2.42: Deadband filter acting on the process measurement signal

deadband filter changes value only if the change of the input signal is larger than the deadband.

Example 2.18 ***Deadband measurement filter in the control loop***

Figure 2.43 shows a simulation of a control system with deadband measurement filter. The process and the PID controller are as described in Example 2.16. In the simulation the measurement noise is a random signal

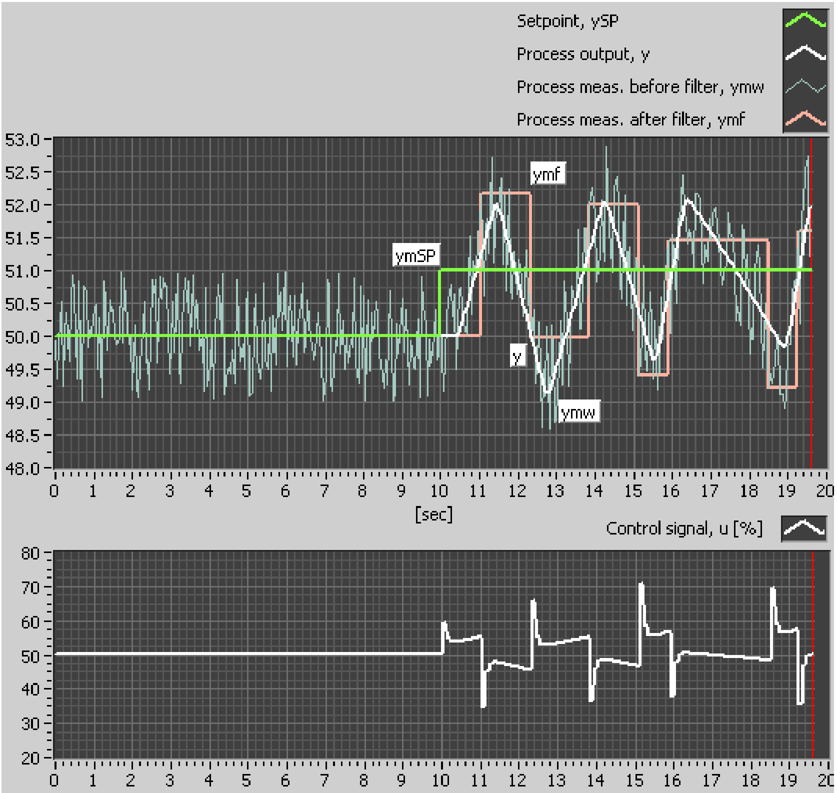

Figure 2.43: Example 2.18: Simulation of a control system with deadband measurement filter

uniformly distributed between −1% and +1%. The deadband of the filter is 2%. The simulation shows the following:

- Up to time $t = 10$s the setpoint is constant (50%). The measurement signal which is the output of the deadband filter is constant since the amplitude of the noise is smaller than the deadband. And since the measurement signal is constant, the control signal generated by the controller is constant – which is good!
- At $t = 10$s the setpoint is changed as a step (from 50% to 51%) which implies that the measurement signal due to the overshoot in

the step response changes value beyond the deadband of 2%. Thereafter the deadband filter acts similar to an on/off-element in the loop, and there are sustained oscillations in the loop – not good!

[End of Example 2.18]

You have in this Section seen two ways of filtering measurement noise:

- *Using a dynamic lowpass filter*: The dynamic filter can be easily tuned via the bandwidth. The filter influences the dynamics and hence the stability of the loop. The controller should be tuned with the filter in the loop.
- *Using a deadband filter*: This filter may give a constant measurement signal, as long as the input signal to the deadband filter does not change more than the deadband of the filter. Once the deadband is exceeded, the deadband filter may behave almost like an on/off controller, causing oscillations in the control loop.

From the above results it seems that the dynamic filter is a safer way than deadband filter to handle measurement noise.

2.8 Performance index of control systems

Assume that different control systems are to be compared, or that different controller parameter for one control system are to be compared. There are several ways to express the performance of the control systems, e.g. bandwidth and stability margins. These measures are based on a mathematical model of the control system, and they are described in Chapter 6.

Alternatively, we can use performance indices which are functions of the observed control error e. These indices does not require a mathematical model. Probably the most frequently used index is the IAE – Integral of Absolute value of control Error [15]:

$$\mathrm{IAE} = \int_0^\infty |e|\, dt \tag{2.84}$$

The less IAE value, the better performance. The IAE value is finite only if e converges towards zero in steady-state, which in practice requires the