Høgskolen i Buskerud / Finn Haugen / 15. mars 2009

Formelliste for eksamen i SEKY3322 Kybernetikk 3

Formlene nedenfor vil bli oppgitt som vedlegg til eksamen 2. april 2009.

På eksamen må du selv velge hvilke(n) av formlene som er aktuelle i den enkelte oppgave.

$$\dot{x}(t_k) \approx \frac{x(t_{k+1}) - x(t_k)}{h} \tag{1}$$

$$\dot{x}(t_k) \approx \frac{x(t_k) - x(t_{k-1})}{h} \tag{2}$$

$$u(t) = u_0 + K_p e(t) + \frac{K_p}{T_i} \int_0^t e \, d\tau + K_p T_d \dot{e}_f(t) \tag{3}$$

$$x(k+1) = f[x(k),\ u(k)] \tag{4}$$

$$y(k) = g[x(k),\ u(k)] \tag{5}$$

$$x(k+1) = Ax(k) + Bu(k) \tag{6}$$

$$y(k) = Cx(k) + Du(k) \tag{7}$$

$$A = \left. \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \right|_{\text{op}} = \left. \frac{\partial f}{\partial x^T} \right|_{\text{op}} \tag{8}$$

$$B = \left. \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix} \right|_{\text{op}} = \left. \frac{\partial f}{\partial u^T} \right|_{\text{op}} \tag{9}$$

$$C = \left. \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{bmatrix} \right|_{\text{op}} = \left. \frac{\partial g}{\partial x^T} \right|_{\text{op}} \tag{10}$$

$$D = \left. \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & \frac{\partial g_1}{\partial u_2} \\ \frac{\partial g_2}{\partial u_1} & \frac{\partial g_2}{\partial u_2} \end{bmatrix} \right|_{\text{op}} = \left. \frac{\partial g}{\partial u^T} \right|_{\text{op}} \tag{11}$$

$$\mathcal{Z}\{y(k)\} = \sum_{k=0}^{\infty} y(k) z^{-k} \tag{12}$$

$$k_1 y_1(z) + k_2 y_2(z) \Longleftrightarrow k_1 y_1(k) + k_2 y_2(k) \tag{13}$$

$$z^{-n}y(z) \Longleftrightarrow y(k-n) \tag{14}$$

$$z^{n}y(z) \Longleftrightarrow y(k+n) \tag{15}$$

$$\text{Unity step at time-step } k=0\text{: } 1 \iff \frac{z}{z-1} \tag{16}$$

$$H_s = \frac{y_s}{u_s} = \lim_{z\to 1} H(z) = H(1) \tag{17}$$

$$L(z) = H_c(z)\underbrace{H_u(z)H_s(z)}_{H_p(z)} = H_c(z)H_p(z) \tag{18}$$

$$T(z) = \frac{L(z)}{1+L(z)} = \frac{\frac{n_L(z)}{d_L(z)}}{1+\frac{n_L(z)}{d_L(z)}} = \frac{n_L(z)}{d_L(z)+n_L(z)} \tag{19}$$

$$m_x = \frac{1}{N}\sum_{k=0}^{N-1} x(k) \tag{20}$$

$$\text{Var}(x) = \frac{1}{N-1}\sum_{k=0}^{N-1} [x(k)-m_x]^2 \tag{21}$$

$$\sigma = \sqrt{\text{Var}(x)} \tag{22}$$

$$R_x(L) = E\{[x(k+L)-m_x][x(k)-m_x]\} \tag{23}$$

$$R_{xy}(L) = E\{[x(k+L)-m_x][y(k)-m_y]\} \tag{24}$$

$$\delta(L) = \begin{cases} 1 \text{ when } L=0 \\ 0 \text{ when } L\neq 0 \end{cases} \tag{25}$$

$$m_y = Gm_v + C \tag{26}$$

$$\sigma_y^2 = G^2\sigma_v^2 \tag{27}$$

$$y = \Phi\theta \tag{28}$$

$$\theta_{\text{LS}} = (\Phi^T\Phi)^{-1}\Phi^T y \tag{29}$$

$$\dot{x}_k \approx \frac{\frac{x_{k+1}-x_k}{h} + \frac{x_k - x_{k-1}}{h}}{2} = \frac{x_{k+1}-x_{k-1}}{2h} \tag{30}$$

$$M_{\text{obs}} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{31}$$

$$x(k+1) = f[x(k),\, u(k)] + Gw(k) \tag{32}$$

$$y(k) = g[x(k),\, u(k)] + Hw(k) + v(k) \tag{33}$$

$$R_w(L) = Q\delta(L) \tag{34}$$

$$R_v(L) = R\delta(L) \tag{35}$$

$$y_p(k) = g\left[x_p(k)\right] \tag{36}$$

$$e(k) = y(k) - y_p(k) \tag{37}$$

$$x_c(k) = x_p(k) + Ke(k) \tag{38}$$

$$x_p(k+1) = f[x_c(k),\, u(k)] \tag{39}$$

$$\dot{x} = f + Bu \tag{40}$$

$$\ddot{x} = f + Bu \tag{41}$$

$$T(s) = \frac{y_{mf}(s)}{y_{m_{SP}}(s)} = \frac{1}{T_C s + 1} e^{-\tau s} \tag{42}$$

$H(s)$	K_p	T_i	T_d
$\frac{K}{s} e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$k_1\left(T_C+\tau\right)$	0
$\frac{K}{Ts+1} e^{-\tau s}$	$\frac{T}{K(T_C+\tau)}$	$\min\left[T,\, k_1\left(T_C+\tau\right)\right]$	0
$\frac{K}{(Ts+1)s} e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$k_1\left(T_C+\tau\right)$	T
$\frac{K}{(T_1 s+1)(T_2 s+1)} e^{-\tau s}$	$\frac{T_1}{K(T_C+\tau)}$	$\min\left[T_1,\, k_1\left(T_C+\tau\right)\right]$	T_2
$\frac{K}{s^2} e^{-\tau s}$	$\frac{1}{4K(T_C+\tau)^2}$	$4\left(T_C+\tau\right)$	$4\left(T_C+\tau\right)$

$$u = B^{-1}\left(K_p e + K_i \int_0^t e d\tau + \dot{r}_{y_f} - f\right) \tag{43}$$

$$u = B^{-1}\left(K_p e + K_i \int_0^t e d\tau + K_d \frac{de_f}{dt} + \ddot{r}_{y_f} - f\right) \tag{44}$$

$$K_{p_p} = K_{p_s}\left(1 + \frac{T_{d_s}}{T_{i_s}}\right) \tag{45}$$

$$T_{i_p} = T_{i_s}\left(1 + \frac{T_{d_s}}{T_{i_s}}\right) \tag{46}$$

$$T_{d_p} = T_{d_s} \frac{1}{1 + \frac{T_{d_s}}{T_{i_s}}} \tag{47}$$

$$x(k+1) = Ax(k) + Bu(k) + G_w w(k) \tag{48}$$

$$J = \sum_{k=0}^{\infty}\left[x^T(k)Qx(k) + u^T(k)Ru(k) + 2x^T(k)Nu(k)\right] \tag{49}$$

$$u(k) = -G(k)x(k) \tag{50}$$

$$M_{\text{control}} = \left[B \vdots AB \vdots A^2 B \vdots \cdots \vdots A^{n-1} B\right] \tag{51}$$

$$
\begin{aligned}
J \;=\; & \sum_{i=N_w}^{N_p} \left[\hat{y}(t_{k+i}|t_k) - r(t_{k+i}|t_k)\right]^T Q \left[\hat{y}(t_{k+i}|t_k) - r(t_{k+i}|t_k)\right] && (52)\\
& + \sum_{i=1}^{N_c-1} \left[\Delta u(t_{k+i}|t_k)\right]^T R \left[\Delta u(t_{k+i}|t_k)\right] && (53)\\
& + \sum_{i=N_w}^{N_p} \left[u(t_{k+i}|t_k) - s(t_{k+i}|t_k)\right]^T N \left[u(t_{k+i}|t_k) - s(t_{k+i}|t_k)\right] && (54)
\end{aligned}
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