9.3 Ratio control and quality and product flow control

9.3.1 Ratio control

The purpose of ratio control is to control a mass flow, say $F_2$, so that the ratio between this flow and another flow, say $F_1$, is

$$F_2 = K F_1$$  \hspace{1cm} (9.25)

where $K$ is a specified ratio which may have been calculated as an optimal ratio from a process model. One example is the calculation of the ratio
between oil inflow and air inflow to a burner to obtain optimal operating condition for the burner. Another example is the nitric acid factory where ammonia and air must be fed to the reactor in a given ratio.

Figure 9.12 shows the structure of ratio control. The setpoint of the flow $F_2$ is calculated as $K$ times the measured value of $F_1$, which is denoted the “wild stream”. The figure shows a control loop of $F_1$. The setpoint of $F_1$ (the setpoint is not shown explicitly in the figure) can be calculated from a specified production rate of the process. The ratio control will then ensure the ratio between the flows as specified.

An alternative way to implement ratio control is to calculate the actual ratio as

$$K_{actual} = \frac{F_2}{F_1}$$

(9.26)

Then $K_{actual}$ is used as a measurement signal to a ratio controller with the specified $K$ as the setpoint and $F_2$ as the control variable, cf. Figure 9.13.

Figure 9.13: An alternative ratio control structure based on measurement of the actual ratio. (FFC = Flow Fraction Controller.)

Although this control structure is logical, it is a drawback that the loop gain, in which $K_{actual}$ is a factor, is a function of the measurements of $F_2$ and $F_1$. Hence, this solution is not encouraged[16].

### 9.3.2 Quality and production rate control

Earlier in this section it was mentioned that the ratio $K$ may origin from an analysis of optimal process operation, say from a specified product quality quantity, say $Q_{SP}$. Imagine however that there are disturbances so
that key components in one of or in both flows $F_1$ or $F_2$ vary somewhat. Due to such disturbances it may well happen that the actual product quality is different from $Q_{SP}$. Such disturbances may also cause the actual product flow to differ from a flow setpoint. These problems can be solved by implementing

- a quality control loop based on feedback from measured quality $Q$ to the ratio parameter $K$, and
- a product flow control loop based on feedback from measured flow $F$ to one of the feed flows.

Figure 9.14 shows the resulting quality and production rate control system.

![Figure 9.14: Control of quality and product flow. (QT = Quality Transmitter. QC = Quality Controller.)](image)

### 9.4 Split-range control

In *split-range control* one controller controls two actuators in different ranges of the control signal span, which here is assumed to be $0 - 100\%$. See Figure 9.15. Figure 9.16 shows an example of split-range temperature control of a thermal process. Two valves are controlled – one for cooling and one for heating, as in a reactor. The temperature controller controls
the cold water valve for control signals in the range 0–50%, and it controls the hot water valve for control signals in the range 50–100%, cf. Figure 9.15.

In Figure 9.15 it is indicated that one of the valves are open while the other is active. However in certain applications one valve can still be open while the other is active, see Figure 9.17. One application is pressure control of a process: When the pressure drop compensation is small (as when the process load is small), valve $V_1$ is active and valve $V_2$ is closed. And when the pressure drop compensation is is large (as when the process load is large), valve $V_1$ is open and valve $V_2$ is still active.

### 9.5 Control of product flow and mass balance in a plant

In the process industry products are created after treatment of the materials in a number of stages in series, which are typically unit processes as blending or heated tanks, buffer tanks, distillation columns, absorbers, reactors etc. The basic control requirements of such a production line are as follows:

- The mass flow of a key component must be controlled, that is, to follow a given production rate or flow setpoint.
Figure 9.16: Split-range temperature control using two control valves

Figure 9.17: In split-range control one valve can be active while another valve is open simultaneously.

- The mass balance in each process unit (tank etc.) must be maintained — otherwise e.g. the tank may go full or empty.

Figure 9.18 shows the principal control system structure to satisfy these requirements. (It is assumed that the mass is proportional to the level.) The position of the production flow control in the figure is just one example. It may be placed earlier (or later) in the line depending on where the key component(s) are added.

Note that the mass balance of an upstream tank (relative to the production flow control) is controlled by manipulating the mass inflow to the tank, while the mass balance of a downstream tank is controlled by manipulating the mass outflow to the tank.
In Figure 9.18 the mass balances are maintained using level control. If the tanks contain vapours, the mass balances are maintained using pressure control. Then pressure sensors (PT = Pressure Transmitter) take the place of the level sensors (LT = Level Transmitters), and pressure controllers (PC = Pressure Controller) take the place of level controllers (LC = Level Controller) in Figure 9.18.

Example 9.7 Control of production line

Figure 9.19 shows the front panel of a simulator of a general production line. The level controllers are PI controllers which are tuned so that the control loops get proper speed and stability (the parameters may be calculated as explained in Chapter 7.2.2). The production flow $F$ is here controlled using a PI controller. Figure 9.19 shows how the level control loops maintain the mass balances (in steady-state) by compensating for a disturbance which is here caused by a change of the production flow. Note that controller LC2 must have negative gain (i.e. direct action, cf. Section 2.6.8) – why?\footnote{Because the process has negative gain, as an increase of the control signal gives a reduction of the level/level measurement.}

[End of Example 9.7]
9.6 Multivariable control

9.6.1 Introduction

Multivariable processes have more than one input variables or one than one output variables. Here are a few examples of multivariable processes:

- A heated liquid tank where both the level and the temperature shall be controlled.
- A distillation column where the top and bottom concentration shall be controlled.
- A robot manipulator where the positions of the manipulators (arms) shall be controlled.
- A chemical reactor where the concentration and the temperature shall be controlled.
• A head box (in a paper factory) where the bottom pressure and the paper mass level in the head box shall be controlled.

To each variable (process output variable) which is to be controlled a setpoint is given. To control these variables a number of control variables are available for manipulation by the controller function.

Multivariable processes can be difficult to control if there are cross couplings in the process, that is, if one control variable gives a response in several process output variables. There are mainly two problems of controlling a multivariable process if these cross couplings are not counteracted by the multivariable controller:

• A change in one setpoint will cause a response in each of the process output variables, not only in the output variable corresponding to the setpoint.

• Assuming that ordinary single loop PID control is used, a controller will “observe” a complicated dynamic system which consists of the multivariable process with all control loops! This can make it difficult to tune each of the PID controllers, and the stability robustness of the control system may be small.

The following sections describe the most common ways to control multivariable processes.

9.6.2 Single loop control with PID controllers

The simplest yet most common way to control a multivariable process is using single loop control with PID controllers. There is one control loop for each process output variable which is to be controlled. The control system structure is shown in Figure 9.20, where subsystems are represented by transfer functions although these subsystems are generally non-linear dynamic systems. Since this process has two control variables and two process output variables, we say that the process is a 2x2 multivariable process.

Pairing of process output variables and control variables

In single loop control of a multivariable process we must determine the pairing of process output variable (its measurement) and control variable
Figure 9.20: Single loop control of a 2x2 multivariable process

(via the PID controller). A natural rule for choosing this pairing is as follows: *The strong process couplings (from control variable to process output variable) should be contained in the control loops.* Following this rule is an effective use of the control variable, and supports stability robustness against variations of the dynamic properties in other parts of the control system. Figure 9.20 shows the correct control system structure if there are strong couplings in $H_{11}(s)$ and in $H_{22}(s)$.

In most cases it is easy to determine the strong pairings. One example is a heated liquid tank where both level and temperature is to be controlled. The two control variables are power supply via a heating element and liquid supply. This process is multivariable with cross couplings since both power supply (control variable 1) and liquid supply (control variable 2) influences both process output variables (level and temperature). (The level is influenced by the power supply through liquid expansion due to
temperature increase.) In this case the process output variable/control variable pairing is obvious: Level ↔ Power and Temperature ↔ Liquid flow, right?5

There are model based methods for analysis of process couplings, as RGA-analysis (Relative Gain Array) and singular value analysis [15].

Controller tuning

In the tuning procedures below you can try the Ziegler-Nichols’ closed loop method or the P-I-D tuning method, cf. Chapter 4.

According to [15] a widely used procedure for tuning the PID controllers in single loop multivariable control is as follows:

Procedure 1:

1. Tune the controller in each of the loops in turn with all the other controllers in manual mode.
2. Close all the loops (set all controllers in automatic mode).
3. If there are stability problems, reduce the gain and/or increase the integral time of the controllers in the least important loops.

An alternative procedure [15] for cases where the control of one specific process variable is more important than the control of other variables is as follows:

Procedure 2:

1. Tune the controller of the most important loop. The other controllers are set in manual mode.
2. Tune the other controllers in sequence, with the tuned controllers set in automatic mode.
3. If there are stability problems, reduce the gain and/or increase the integral time of the controllers in the least important loops.

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5Wrong :-)}
Example 9.8 *Single loop multivariable control*

See Figure 9.20. The process transfer functions are on the form

\[
H_{ij}(s) = \frac{y_i(s)}{u_j(s)} = \frac{K_{ij}}{T_{ij}s + 1} e^{-\tau_{ij}s} \tag{9.27}
\]

with these parameters:

- \(K_{11} = 1; T_{11} = 1; \tau_{11} = 0.5\) \hspace{1cm} (9.28)
- \(K_{12} = 0.5; T_{12} = 1; \tau_{12} = 0.5\) \hspace{1cm} (9.29)
- \(K_{21} = 0.5; T_{21} = 1; \tau_{21} = 0.5\) \hspace{1cm} (9.30)
- \(K_{22} = 1; T_{22} = 1; \tau_{22} = 0.5\) \hspace{1cm} (9.31)

Thus, there are cross couplings “both ways” in the process since both \(K_{12}\) and \(K_{21}\) are different from zero.

The measurement transfer functions are \(H_{m1}(s) = 1 = H_{m2}(s)\). The controllers are PID controllers tuned according to Procedure 1 described above (with the Ziegler-Nichols’ closed loop method). The tuning gives

- \(K_{p1} = 2.0; T_{i1} = 0.9; T_{d1} = 0.23\) \hspace{1cm} (9.32)
- \(K_{p2} = 2.0; T_{i2} = 0.9; T_{d2} = 0.23\) \hspace{1cm} (9.33)

However, simulations shows that the multivariable control system actually is unstable using the above PID settings. So, re-tuning was necessary. Decreasing the proportional gains from 2.0 to 1.4 was sufficient in this case. The final settings are

- \(K_{p1} = 1.4; T_{i1} = 0.9; T_{d1} = 0.23\) \hspace{1cm} (9.34)
- \(K_{p2} = 1.4; T_{i2} = 0.9; T_{d2} = 0.23\) \hspace{1cm} (9.35)

Figure 9.21 shows simulated responses in \(y_{1m}\) and \(y_{2m}\) due to a step in \(y_{1SPm}\). As expected, the setpoint step gives a cross response in \(y_{2m}\). The stability of the control system seems to be acceptable.

[End of Example 9.8]
Figure 9.21: Example 9.8: Single loop multivariable control. Simulated responses in $y_{1m}$ and $y_{2m}$ due to a step in the setpoint $y_{1SPm}$.

### 9.6.3 Single loop PID control combined with decoupling

If the interaction between the control loops in a multivariable control system is problematic (as if the cross responses are too large or if there are stability problems), you may consider using a **decoupler** together with the PID controllers. With decoupling the controller counteracts the cross couplings in the process so that there are no interaction between the control loops. Thus, the control loops are decoupled, but the process cross couplings are still there, of course.

Several methods of decoupled control exist. Here a method called **linear decoupling** is described. Figure 9.22 shows a block diagram of a multivariable control system with decoupling. The controller consists of two parts:

- A decoupler in series with the process. The purpose of the decoupler
Figure 9.22: Multivariable control system with decoupler and single loop PID-controllers

is to counteract the cross couplings in the process.

- A controller consisting of two independent PID controllers which controls the combination of decoupler and process.

The decoupler transfer functions $D_1(s)$ and $D_2(s)$ must be designed so that the cross couplings via $H_{12}(s)$ and $H_{21}(s)$ in the process are counteracted. If this is achieved, each of the PID controllers sees just one monovariable process, and there will be no interacting control loops. $z_1$ and $z_2$ is transformed control variables for the “new” decoupled process. The physical control variables are however still $u_1$ and $u_2$.

Let us derive the decoupler transfer functions $D_1(s)$ and $D_2(s)$. We start with $D_1(s)$. It will be derived from the requirement that the net effect that $z_1$ has on $y_1$ is zero. Mathematically, cf. the block diagram in Figure 9.22, we require:

$$D_1(s)H_{22}(s)z_1(s) + H_{21}(s)z_1(s) = 0 \quad (9.36)$$
for all values of $z_1$. This is satisfied with

$$D_1(s) = -\frac{H_{21}(s)}{H_{22}(s)}$$

(9.37)

Similarly, $D_2(s)$ becomes

$$D_2(s) = -\frac{H_{12}(s)}{H_{11}(s)}$$

(9.38)

**Example 9.9 Decoupling**

Assume given the same process model as in Example 9.8. (9.37) and (9.38) becomes

$$D_1(s) = -\frac{K_{21} T_{21} e^{-\tau_{21}s}}{T_{22}s + 1} = -\frac{K_{21} T_{22}s + 1}{K_{22} T_{21} s + 1} e^{(\tau_{22}-\tau_{21})s} = -0.5$$

(9.39)

$$D_2(s) = -\frac{K_{12} T_{11}s + 1}{K_{11} T_{12}s + 1} e^{(\tau_{11}-\tau_{12})s} = -0.5$$

(9.40)

The PID controllers are tuned with the Ziegler-Nichols’ closed loop method (with the decoupler in action):

$$K_{p_1} = 2.7; T_{i_1} = 0.9; T_{d_1} = 0.23$$

(9.41)

$$K_{p_2} = 2.7; T_{i_2} = 0.9; T_{d_2} = 0.23$$

(9.42)

Figure 9.23 shows simulated responses in $y_{1m}$ and $y_{2m}$ due to a step in the setpoint $y_{1s_{m}}$. Ideally there is no cross response in $y_{2m}$.6

[End of Example 9.9]

### 9.6.4 Model-based predictive control

**Model-based predictive control** or MPC has become an important control method, and it can be regarded as the next most important control method in the industry, next to PID control. Commercial MPC products are available as separate products or as modules included in automation products. MPC can be applied to multivariable and non-linear processes. The controller function is based on a continuous calculation of the optimal

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6The simulation shows a (very) small response in $y_{2m}$, but this is due to imperfect numerical conditions in the simulator.
Figure 9.23: Example 9.9: Single loop PID-control with decoupling: Simulated responses in $y_{1m}$ and $y_{2m}$ due to a step in the setpoint $y_{1SP_m}$.

sequence or time-series of the control variable. The optimization criterion which is to be minimized is typically stated as follows:

$$J = \sum_{j=1}^{N} \left\{ [y_{SP}(t_{k+j}) - y(t_{k+j}|t_k)]^2 + \lambda(j)[u(t_{k+j-1})]^2 \right\}$$

(9.43)

where $y_{SP}$ is the setpoint, $y$ is the process output variable and $u$ is the control variable. In general these variables can be vectors. $\lambda$ is a weight function. $t_k$ is the present time. $N$ is the prediction horizon, a number of future time steps over which $J$ is defined. This optimization criterion defines the criterion of “good control”: The less value of $J$, the better control.

The optimization criterion must have a constraint, which is the process model including physical constraints of the control variable and the state variables. The process model form are one of the following (different MPC
implementations may assume different model forms):

- Impulse response model (which can be derived from simple experiments on the process)
- Step response model (same comment as above)
- Transfer function model (which is a general linear dynamic model form than the impulse response model and the step response model)
- State-space model (which is the most general model form since it may include nonlinearities and it may be valid over a broad operating range)

Figure 9.24 illustrates how predictive control works.

![Figure 9.24: How predictive control works](image)

Predictive control is based on the following calculations, which are executed at each time step:
1. The (future) control signal sequence \( u(t_{k+j-1}|t_k) \) for \( j = 1, \ldots, N \), that is, \( u(t_k), u(t_{k+1}), \ldots, u(t_{k+N-1}) \), is calculated to be the control variable sequence which minimizes \( J \) (the sequence is therefore optimal). Terms as \( u(t_{k+L}|t_k) \) gives the value of \( u \) for time \( t_{k+L} \) calculated from data available at time \( t_k \). The same applies to \( y(t_{k+L}|t_k) \), which is calculated (predicted) using the process model.

2. Of the optimal control signal sequence, the first term, \( u(t_k|t_k) \), is used to actually control the process.

3. At the next time step the points above are repeated.

Above it was assumed that the MPC-controller calculates directly the control variable to the process. However the MPC-controller may also calculate setpoints for local PID controllers, see Figure 9.25. This structure ensures that the process can still be controlled with conventional PID controllers in periods when the MPC-controller is inactive (due to configuration or maintenance). Using PID controllers locally may enhance operation safety.