

Lecture notes in
MODELS, ESTIMATION and CONTROL

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Preface

This book covers estimation of model parameters and states and model-based control, and the systems theory required. I have selected the topics among many possible topics by considering what I assume are the most relevant topics in an application oriented course about these topics.

It is assumed that you – the reader – has basic knowledge about complex number, differential equations, and Laplace transform based transfer functions (s -transfer functions), and also that you have knowledge about basic control methods, i.e. PID control, feedforward control and cascade control. A reference for these topics is my book *Basic Dynamics and Control* [6].

Supplementary material, as tutorials, ready-to-run simulators, and instructional videos are available at techteach.no.

This book is available for sale only from the web site <http://techteach.no>.

It is not allowed to make copies of the book.

If you want to know about my background, please visit my home page techteach.no/adm/fh.

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