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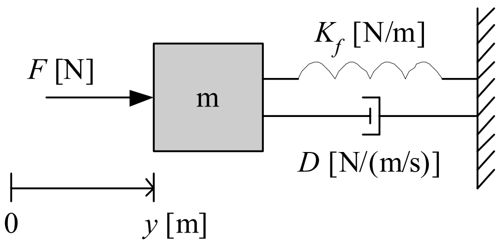
SEKY3322 Kybernetikk 3: Tilstandsrommodell for masse-fjær-demper

Problem 1

Figure 1 shows a mass-spring-damper-system. (One concrete example is



Figur 1: Mass-spring-damper

the wheel suspension system on a car.) We assume that the spring force is zero when position y is zero. Force balance yields

$$\begin{aligned} m\ddot{y}(t) &= F(t) - F_d(t) - F_f(t) \\ &= F(t) - D\dot{y}(t) - K_f y(t) \end{aligned} \tag{1}$$

1. Find a contonuous-time state-space model for the system with state variables x_1 = position and x_2 = speed. The output variable is y = position. Also, set $u = F$. It is not required to write this state-space model on matrix-vector form.

2. Find a corresponding discrete-time state-space model based in discretization with the Forward difference method. It *is* required to write this state-space model on matrix-vector form.

Solution to Problem 1

1. The model (1) can be written as the following equivalent set of two first order differential equations:

$$\dot{x}_1 = x_2 \tag{2}$$

$$m\dot{x}_2 = -Dx_2 - K_f x_1 + u \tag{3}$$

which can be written on the state-space model form as:

$$\dot{x}_1(t) = x_2(t) \tag{4}$$

$$\dot{x}_2(t) = \frac{1}{m}\left[-Dx_2(t) - K_f x_1(t) + u(t)\right] \tag{5}$$

$$y(t) = x_1(t) \tag{6}$$

2. Substituting the time-derivatives with the Forward difference approximation gives (with $k \triangleq t_k$)

$$\frac{x_1(k+1) - x_1(k)}{h} = x_2(k) \tag{7}$$

$$\frac{x_2(k+1) - x_2(k)}{h} = \frac{1}{m}\left[-Dx_2(k) - K_f x_1(k) + u(k)\right] \tag{8}$$

which can be written on state-space model form as

$$x_1(k+1) = x_1(k) + hx_2(k) \tag{9}$$

$$x_2(k+1) = x_2(k) + h\frac{1}{m}\left[-Dx_2(k) - K_f x_1(k) + u(k)\right] \tag{10}$$

$$= -\frac{hK_f}{m}x_1(k) + \left(1 - \frac{hD}{m}\right)x_2(k) + \frac{h}{m}u(k) \tag{11}$$

Discretizing (6) gives

$$y(k) = x_1(k) \tag{12}$$

Writing (9) – (10) and (12) on matrix-vector form is

$$\underbrace{\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} 1 & h \\ -\frac{hK_f}{m} & \left(1 - \frac{hD}{m}\right) \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 0 \\ \frac{h}{m} \end{bmatrix}}_{B} u(k) \tag{13}$$

$$y(k) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(k) \tag{14}$$