**Exercise 1** ***Chip tank***

Figure 1 shows a chip tank with a feed screw and conveyor belt (the belt has constant speed).[1] There is an outflow of chip via an outlet at the

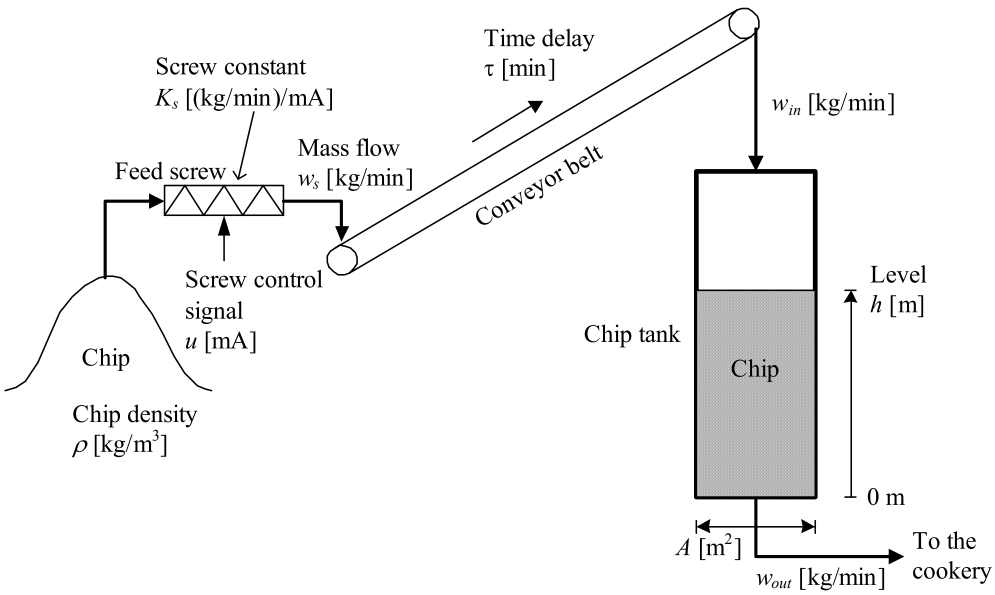


Figure 1: Exercise 1: Chip tank with a feed screw and conveyor belt

bottom of the tank. The mass flow $w_s$ from the feed screw to the belt is proportional to the screw control signal $u$:

$$w_s = K_s u \tag{1}$$

The mass flow $w_{in}$ into the chip tank is equal to $w_s$ but time delayed time $\tau$:

$$w_{in}(t) = w_s(t - \tau) \tag{2}$$

1. Develop a mathematical model of the chip level.
2. Draw an input-output block diagram of the system. Define the input and output variables, but it is assumed that the level is of particular interest.

**Exercise 2** ***Heated tank***

Figure 2 shows a heated tank with liquid flow. Both the liquid in the tank

[1] Typically, there is such a chip tank in the beginning of the production line of a paper mass factory.

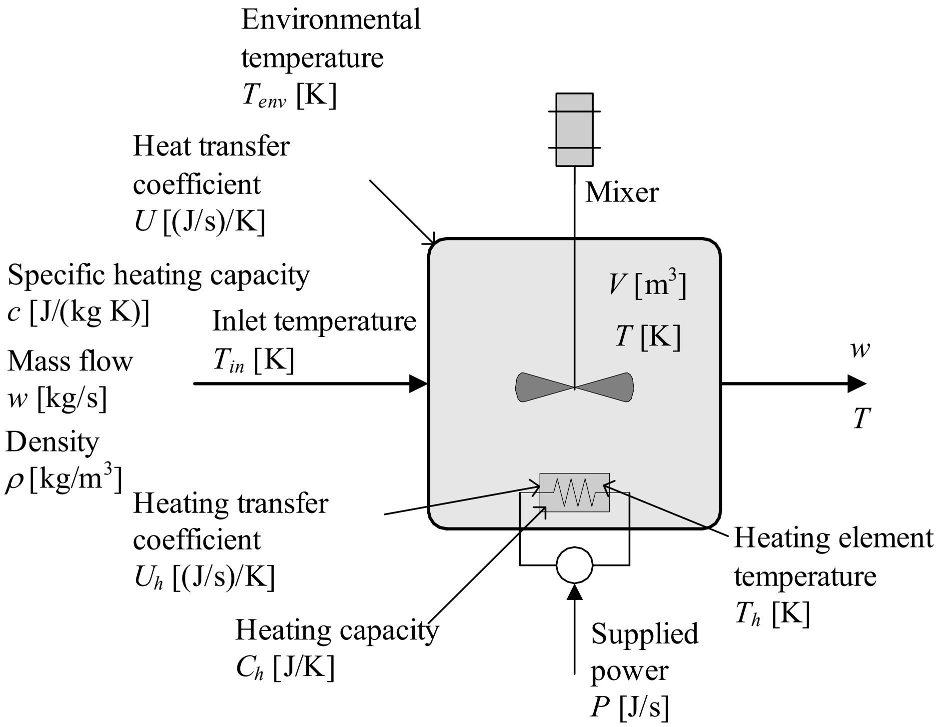


Figure 2: Exercise 2: Heated tank with liquid flow

and the heating element have heat capacities. The heat transfer between the environment and the liquid is proportional to the temperature difference between them. The heat transfer between the heating element and the liquid is also proportional to the temperature difference between them. Assume homogenous conditions.

1. Develop a mathematical model of the temperatures of the liquid in the tank and the heating elementet.

2. Draw an input-output block diagram of the system. Consider then $P$, $T_o$, $T_i$ and $w$ as input variables and $T$ as an output variable.

**Exercise 3** ***Ship model***

Figure 3 shows a ship. In this exercise we concentrate on the forward direction (i.e., the movements in the other directions are disregarded). The wind acts on the ship with the force $F_w$. The hydrodynamic damping force $F_h$ (damping from the water) is proportional to the square of the difference between the ship speed $u$ and the water current speed $u_c$. Assume that the proportionality constant is $D_x$.

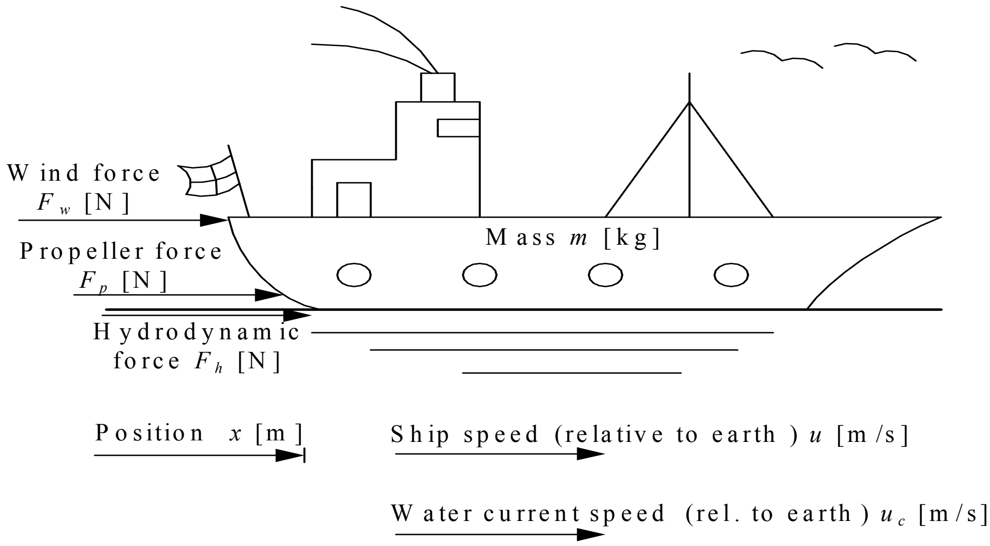


Figure 3: Exercise 3: A ship

1. What is the mathematical relation between speed $u$ and position $x$?
2. Develop a mathematical model of the ship expressing the motion (the position) in the forward direction.
3. Draw an input-output block diagram of the system. Assume that the ship position is the variable of particular interest.

**Solution 1**

1. Since there is a time delay in the system (due to the transport delay of the conveyor belt) it is important to inlcude the time argument in the equations. The mass balance if the chip contents of the tank is

$$\begin{aligned}\frac{d}{dt}\left[\rho A h(t)\right] = \underline{\underline{\rho A \dot{h}(t)}} &= w_{in}(t) - w_{out}(t) \\ &= w_s(t-\tau) - w_{out}(t) \\ &= \underline{\underline{K_s u(t-\tau) - w_{out}(t)}}\end{aligned} \quad (3)$$

2. Figure 4 shows the block diagram.

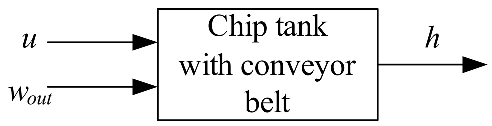


Figure 4: Solution 1: Block diagram of chip tank with conveyor belt

**Solution 2**

1. Energy balance of the liquid in the tank:

$$\underline{\underline{\frac{d(c\rho V T)}{dt} = c\rho V \dot{T} = cwT_i - cwT + U(T_o - T) + U_h(T_h - T)}} \quad (4)$$

Energy balance of the heating element:

$$\underline{\underline{\frac{d(CT_h)}{dt} = C\dot{T}_h = P + U_h(T - T_h)}} \quad (5)$$

2. Figure 6 shows the block diagram.

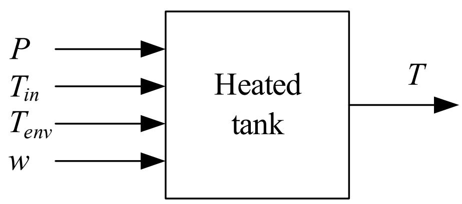


Figure 5: Solution 2: Block diagram of the heated tank

**Solution 3**

1. The relation between position $x$ and speed $u$ is

$$\underline{\underline{\dot{x} = u}} \tag{6}$$

2. Force balance:

$$\underline{\underline{m\dot{u}}} = F_p + F_h + F_w \tag{7}$$

$$= \underline{\underline{F_p + D_x|u - u_c|\,(u - u_c) + F_w}} \tag{8}$$

(6) and (8) constitutes the model.

Alternatively, since

$$\dot{u} = \ddot{x} \tag{9}$$

the model can be expressed as

$$\underline{\underline{m\ddot{x} = F_p + D_x|\dot{x} - u_c|\,(\dot{x} - u_c) + F_w}} \tag{10}$$

3. We can regard $F_p$, $F_w$ and $u_c$ as input variables, and $x$ as the output variable. Figure 6 shows the block diagram.

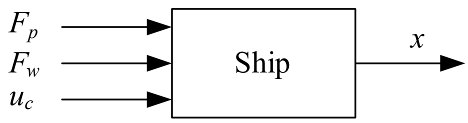


Figure 6: Solution 3: Block diagram of the ship