Exercise 1 ***Block diagram of a differential equation model***

The thermal system described in Example 3 side 23 etc. in the text book (Dynamic Systems) has the following mathematical model:

$$c\rho V\dot{T}(t) = P(t) + cw\left[T_i(t) - T(t)\right] \tag{1}$$

The initial state is $T(0)$.

1. Consider T as output variable, and P and T_i as input variables. Draw a mathematical block diagram for (1). (You may consider the terms $c\rho V$ and cw as being constant parameters.)
2. Now consider w as an input variable (in addition to P and T_i). Draw a mathematical block diagram for (1).

Exercise 2 ***Transfer function of a simple differential equation***

Calculate the transfer function $H(s)$ from u to x for the following differential equation:

$$a_1\dot{x}(t) + a_0 x(t) = bu(t) \tag{2}$$

Exercise 3 ***The transfer function of a nonlinear modell***

Given the following model of a liquid tank:

$$\dot{x}(t) = \frac{K_p}{A_t}u(t) - \frac{K_u}{A_t}\sqrt{\rho g x(t)} \tag{3}$$

where x is the level and u is the inlet pump control signal. Try to calculate the transfer function from u to x for this model.

Exercise 4 ***Transfer function of a wood chip tank with conveyor belt***

According to the solution of Exercise **??** the mathematical model of a wood chip tank with conveyor belt is:

$$\rho A\dot{h}(t) = K_s u(t-\tau) - w_{out}(t) \tag{4}$$

Calculate the transfer function $H_1(s)$ from the control signal u to the level h and the transfer function $H_2(s)$ from the outflow w_{out} to the level h.

Solution 1

1. We start by expressing the solution $T(t)$ of the differential equation (1). The state-space model form is:

$$\dot{T} = \frac{1}{c\rho V}\left[P + cw\left(T_i - T\right)\right] \tag{5}$$

By integrating this model we get

$$T(t) = T(0) + \int_0^t \underbrace{\frac{1}{c\rho V}\left[P + cw\left(T_i - T\right)\right]}_{\dot{T}(\theta)} d\theta \tag{6}$$

Figure 1 shows the block diagram corresponding to this (integral) equation.

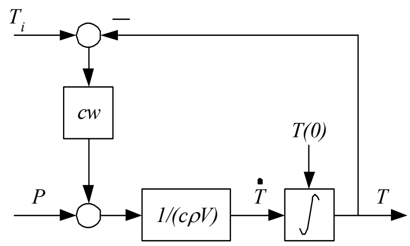

Figure 1: Solution 11: The block diagram

2. Figure 2 shows the block diagram.

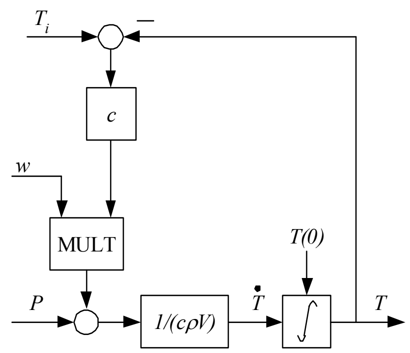

Figure 2: Solution 12: Block diagram

Solution 2

Taking the Laplace transform of the differential equation yields

$$a_1\left[sx(s) - x_0\right] + a_0 x(s) = bu(s) \tag{7}$$

Solving for $x(s)$:

$$x(s) = \frac{a_1}{a_1 s + a_0} x_0 + \underbrace{\frac{b}{a_1 s + a_0}}_{H(s)} u(s) \tag{8}$$

The transfer function from u to x becomes

$$\underline{\underline{H(s) = \frac{b}{a_1 s + a_0}}} \tag{9}$$

Solution 3

We can *not* calculate the transfer function from u to x for the given model because the model is nonlinear (due to the square root function). (However, a transfer function can be derived from a linearized version of the model.)

Solution 4

Taking the Laplace transform of (4) yields

$$\rho A\left[sh(s) - h_0\right] = K_s e^{-\tau s} u(s) - w_{out}(s) \tag{10}$$

where $e^{-\tau s}$ is due to the Laplace transform property (B.3) in the text-book. Manipulating (10) yields

$$h(s) = \frac{1}{\rho A s} h_0 + \underbrace{\frac{K_s}{\rho A s} e^{-\tau s}}_{H_1(s)} u(s) + \underbrace{\left(-\frac{1}{\rho A s}\right)}_{H_2(s)} w_{out}(s) \tag{11}$$

Thus, the transfer functions are

$$\underline{\underline{H_1(s) = \frac{K_s}{\rho A s}}} \tag{12}$$

and

$$\underline{\underline{H_2(s) = -\frac{1}{\rho A s}}} \tag{13}$$